Math 5111, Real Analysis, Spring 2018 — Homework #9

Due In Class: Wednesday, April 18, 2018

Reading: Read 11.2, 3.1–3.2, 3.4.

Turn in the following problems. Exercise a.b.c refers to Exercise c at the end of Section a.b of the textbook. You are welcome (even encouraged) to work on problem sets with other students, but ultimately should write up your final solutions independently.

Problem AQ: Exercise 11.2.10

Problem AR: Exercise 11.2.14

Problem AS: Recall that a measure μ on $(X, \mathcal{M})$ is *carried by* $\mathcal{N} \subset \mathcal{M}$ if there exists countably many $N_i \in \mathcal{N}$ such that $\mu(X \setminus \bigcup_i N_i) = 0$; μ is *singular to* $\mathcal{N}$ if $\mu(N) = 0$ for all $N \in \mathcal{N}$. Moreover, we proved in class that if μ is σ-finite, then μ can be uniquely written as

$$\mu = \mu_{\mathcal{N}} + \mu_{\mathcal{N}}^{\perp},$$

where $\mu_{\mathcal{N}}$ and $\mu_{\mathcal{N}}^{\perp}$ are carried by and singular to $\mathcal{N}$, respectively.

Assume that μ, ν are σ-finite measures on $(X, \mathcal{M})$, $\mu_{\mathcal{N}} = \mu \llcorner A$, and $\mu_{\mathcal{N}}^{\perp} = \mu \llcorner X \setminus A$ for some set $A \in \mathcal{M}$ that is a countable union of sets in $\mathcal{N}$. Prove that if $\nu \ll \mu$, then

$$\nu_{\mathcal{N}} = \nu \llcorner A \quad \text{and} \quad \nu_{\mathcal{N}}^{\perp} = \nu \llcorner A.$$

Problem AT: *Let μ be a σ-finite measure on $(X, \mathcal{M})$. Prove that there exists a finite measure ρ on $(X, \mathcal{M})$ with $\rho(X) = 1$ such that $\mu \ll \rho \ll \mu$.*

Problem AU: Exercise 3.2.10