Math 4110/5110, Introduction to Modern Analysis, Fall 2015 — Homework #5

Due In Class: Thursday, October 22

Reading: Read Chapter 3 and 4.

Do the following problems.

Problem A: Exercise 3.23

Problem B: Exercise 3.24(b,c) (You may assume 3.24(a) without proof.)

Problem C: Exercise 3.24(d,e) (You may assume 3.24(a,b,c) without proof.)

Problem D: Let X be the metric space of nonempty closed subsets of $S = [0,1]^2$, equipped with the Hausdorff distance. Prove that X is totally bounded, i.e. for all $r > 0$ there exist finitely many balls of radius $r > 0$ in X that cover X.

Problem E: Let X be the metric space of nonempty closed subsets of $S = [0,1]^2$, equipped with the Hausdorff distance. Prove that X is complete, i.e. every Cauchy sequence in X converges in X.
Hint: Suppose that $(K_n)_{n=1}^\infty$ is a Cauchy sequence in X. Prove that $K_n \to K$, where

$$K = \left\{ x \in S : x = \lim_{i\to\infty} x_{n_i} \text{ where } x_{n_i} \in K_{n_i} \text{ for some subsequence } (K_{n_i})_{i=1}^\infty \text{ of } (K_n)_{n=1}^\infty \right\}.$$

Remember to check that $K \in X$.

Problem F: Let X be the metric space on nonempty closed subsets of $S = [0,1]^2$, equipped with the Hausdorff distance. Prove that if $(K_n)_{n=1}^\infty$ is a sequence in X, then there exists $K \in X$ and a subsequence $(K_{n_i})_{i=1}^\infty$ of $(K_n)_{n=1}^\infty$ such that $\lim_{i\to\infty} K_{n_i} = K$.
Remark: Your proof should be short.