Due In Class: Tuesday, October 13

Reading: Read Chapter 3.

Do the following problems.

Problem A: Exercise 2.30.

Problem B: Let $(\mathcal{CB}, \mathrm{HD})$ denote the metric space of nonempty compact subsets of the plane $\mathbb{R}^2$, equipped with the Hausdorff distance. Prove that $(\mathcal{CB}, \mathrm{HD})$ is separable.

Problem C: Exercise 3.1.

Problem D: Using the definition of the limit, prove that if $(a_n)_{n=1}^{\infty}$ is a sequence of nonnegative real numbers and $a = \lim_{n\to\infty} a_n$ exists, then $\lim_{n\to\infty} \sqrt{a_n} = \sqrt{a}$.

Problem E: Exercise 3.2. *You must prove your assertions.*

Problem F: Let X and Y be metric spaces, let $f : X \to Y$ be Lipschitz (i.e. there exists $L \in [0, \infty)$ such that $\mathrm{dist}_Y(f(x_1), f(x_2)) \leq L\,\mathrm{dist}_X(x_1, x_2)$ for all $x_1, x_2 \in X$. Prove that if $(x_n)_{n=1}^{\infty}$ is a convergence sequence in X, then $(f(x_n))_{n=1}^{\infty}$ is a convergent sequence in Y.