Math 4110/5110, Introduction to Modern Analysis, Fall 2015 — Homework #2

Due In Class: Thursday, September 17

Reading: If you haven't done so already, please take a glance at *Some Remarks on Writing Mathematical Proofs* by John M. Lee, available at

`http://www.math.washington.edu/~lee/Writing/writing-proofs.pdf`

Start reading Chapter 2 in the textbook.

Prove the following statements.

Theorem 1. *Let X and Y be arbitrary sets. A function $f : X \to Y$ is surjective if and only if f has a right inverse g, i.e. a function $g : Y \to X$ such that $f(g(y)) = y$ for all $y \in Y$.*

Theorem 2 (Exercise 2.2)**.** *The set of algebraic complex numbers is countable.*

Theorem 3 (Exercise 2.4)**.** *The set of irrational real numbers is uncountable.*

Theorem 4 (Exercise 2.7)**.** *Let $A_1, A_2, \ldots$ be subsets of a metric space. Then:*

(a) *For all $n \geq 1$, $\overline{\bigcup_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$.*
(b) *The set $\overline{\bigcup_{i=1}^{\infty} A_i} \supseteq \bigcup_{i=1}^{\infty} \overline{A_i}$.*
(c) *The previous inclusion may be strict.*

Theorem 5 (Exercise 2.8)**.** *Let $U \subset \mathbb{R}^2$ be an open set. Every point of U is an accumulation point of U.*

Theorem 6. *Every closed ball in a metric space (i.e. a set of the form $\overline{B}(x, r) = \{y : \operatorname{dist}(x, y) \leq r\}$) is a closed set.*