Math 4110/5110, Introduction to Modern Analysis, Fall 2014 Homework #7

Due In Class: Thursday, October 23.

Reading: Read Chapter 4.

Turn in the following problems. Exercise a.b refers to Exercise b in Chapter a of the textbook.

Problem A: Exercise 4.7

Problem B: Exercise 4.8

Problem C: Exercise 4.11 (the first part only): Prove that if $f : X \to Y$ is uniformly continuous and $(x_n)_{n=1}^{\infty}$ is Cauchy in X, then $(f(x_n))_{n=1}^{\infty}$ is Cauchy in Y.

Problem D: Exercise 4.12

Problem E: Exercise 4.20: Nowadays the 'distance to E function' is usually denoted by $\operatorname{dist}(\cdot, E) : X \to [0, \infty)$, where $\operatorname{dist}(x, E) = \inf_{z \in E} d(x, z)$ for all $x \in X$.

The following problems should be included in the writing portfolio, a draft of which is due in class on Thursday, November 13th. The portfolio should be typeset (e.g. using LaTeX, etc.) and should follow the conventions outlined in 'Some Remarks on Writing Mathematical Proofs' by John M. Lee (available by link on the course webpage). You may discuss the solutions of portfolio problems with your classmates, but all write-ups should be completed independently. Feedback on your portfolio is available at any time during the instructor's office hours.

Portfolio Problem 1: Give a complete, self-contained proof of Theorem 4.9 in the textbook.

Portfolio Problem 2: An extended real-valued function $\eta : [0, +\infty) \to [0, +\infty]$ is called a *modulus of continuity* if $\eta(0) = 0$ and $\lim_{t \to 0} \eta(t) = 0$. A function $f : X \to Y$ between metric spaces *admits a modulus of continuity inequality* if there is a modulus of continuity η such that

$$d_Y(f(x), f(x')) \leq \eta\left(d_X(x, x')\right) \quad \text{for all } x, x' \in X.$$

Prove that $f : X \to Y$ is uniformly continuous if and only if f admits a modulus of continuity inequality.