All problem sets are optional. If you turn in solutions to problems, I will read and return them to you with feedback.

Problem G: Let $E \subset \mathbb{R}^2$ be the four-corner Cantor set, defined by starting with the unit square $[0,1]^2$, dividing into 16 equal size subsquares, deleting all but the four corner squares, and iterating. For this problem you may assume without proof that $\mathcal{H}^1(E) = \sqrt{2}$.

(1) Prove that

$$\liminf_{r \downarrow 0} \frac{\mathcal{H}^1(E \cap B(0,r))}{2r} < \limsup_{r \downarrow 0} \frac{\mathcal{H}^1(E \cap B(0,r))}{2r}.$$

(2) Find upper and lower bounds on the quantities in part (1).

Problem H (Challenging): Let E be the four-corner Cantor set. Again, you may assume that $\mathcal{H}^1(E) = \sqrt{2}$. Prove that at $\mathcal{H}^1$-a.e. $x \in E$,

$$\liminf_{r \downarrow 0} \frac{\mathcal{H}^1(E \cap B(x,r))}{2r} < 1.$$

Problem I: Let $f : \mathbb{R}^n \to \mathbb{R}^m$ satisfy the Hölder inequality $|f(x) - f(y)| \leq H|x-y|^{1/s}$ for some $H < \infty$ and $s \geq 1$. Given $t \geq 0$ and $A \subset \mathbb{R}^n$ satisfying $0 < \mathcal{H}^t(A) < \infty$, find the best possible bounds on the Hausdorff dimension and corresponding Hausdorff measure of $f(A)$.